31° SIICUSP

Exploring Big Bang toy models based on N-body classical newtonian mechanics

Octavio Augusto Potalej

Eduardo Colli

IME/USP

oapotalej@usp.br

Objectives

The evolution of gravitational systems is complex, and numerically simulating the N-body problem is one way to study them. There are several numerical integration methods, including those that preserve the symplectic structure of the system and those that require numerical corrections during evolution.

The toy model has properties that allow discussing dynamic characteristics of the universe, including an alternative of arrows of time offered by Shape Dynamics from Mach-Poincare Principle, which is not based on entropy but on distances between particles of the system.

We studied the initial conditioning of the problem, integration methods, numerical corrections and tested some of the predictions from Shape Dynamics.

Materials and Methods

The basic theory of N-body problem was consulted in [1]. The model used has initial conditions such that all first integrals of the system (momenta and energy) are vanished, as proposed by [2] and [3]. The simulations were developed in Python [4] and Fortran [5]. The numerical integration methods Runge-Kutta fourth-order (RK4) and Velocity Verlet (VV), were implemented, studied from [6] and [7]. The Runge-Kutta-Fehlberg (RKF45) method and a conservative integration method proposed by [8] were also studied and tested, which were not used in the simulations due to their technical limitations. A numerical correction was developed from the first order condition by Karush-Kuhn-Tucker (KKT) according to [9], to obtain greater accuracy with RK4. To avoid singularities, perfectly elastic collisions were implemented.

Results

The use of RK4 without numerical correction leads do implausible solutions for the given initial conditions, as the trajectories obtained distance themselves from the level curves defined in the phase space by the first integrals, as in figure 1.

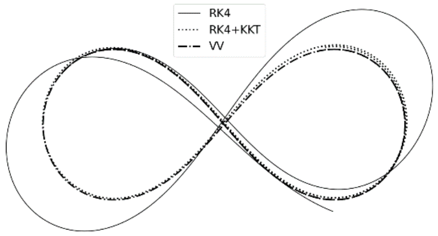

Figure 1: One of trajectories in a 3-body problem via RK4, RK4 with KKT correction and VV.

The application of KKT-based corrector showed greater accuracy than the RK4 and VV methods, with the exception of cases in which the correction reduce the distance between already close bodies, destabilizing the integration. In such cases, the elastic collisions preserved the initial symplectic structure, although their use generates a different dynamical system than the one without collisions. Even so, it was an interesting case study in which it was possible to observe that the predictions by [2], presented below, are maintained.

Two dispersion measures defined in the N-body problem are potential energy V, dominated by shortest distance between the bodies, and the moment of inertia I, dominated by the greatest distance, given by:

$$V = -\sum_{a<b} \frac{G m_a m_b}{r_{ab}}, \qquad I = \sum_{a<b} \frac{m_a m_b r_{ab}^2}{M}, \qquad (1)$$

where $r_{ab} = \|r_a - r_b\|$ and $M = \sum_{a=1}^{N} m_a$, with $m_a \in \mathbb{R}, r_a, p_a \in \mathbb{R}^3, 1 \leq a \leq N$, respectively, the masses, positions and linear momenta of the bodies. The Lagrange-Jacobi Relation, given by:

$$\ddot{I} = 4E - 2V, \qquad (2)$$

where E is the total energy, indicates that the moment of inertia has concave upwards if $E \geq 0$, and its minimum point being called the *Janus Point.* Defining:

$$D = \frac{1}{2}\dot{I} = \sum_{a=1}^{N} \langle r_a, p_a \rangle \qquad (3)$$

as the *dilatational momentum*, there is a monotonic function such that $D = 0$ corresponds to the Janus Point and, therefore, can be used as a measure of the system's evolution. This can be done with the normalization:

$$\sigma_a = \sqrt{\frac{m_a}{I}}\, r_a, \qquad \pi_a = \sqrt{\frac{I}{m_a}}\, p_a - D\sigma_a, \qquad (4)$$

where σ_a is the *shape coordinates* and π_a is the *shape momenta*. In the new coordinates, which are dimensionless, the motion is limited to a unit sphere.

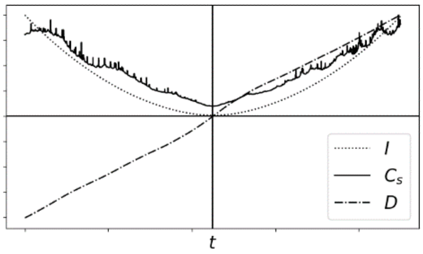

Figure 2: Evolution of a 100-body problem.

It is also possible to obtain a dimensionless dispersion measure that distinguishes the turning point called *complexity*, given by:

$$C_S = |V|\sqrt{I}M^{-2}. \qquad (5)$$

Identifying the arrows of time as a direction in which complexity grows, the symmetry around the Janus Point provides a common past and two possible futures, with similar evolutions, as can be seen in figure 2. Furthermore, it is impossible to identify in which of futures a system is evolving.

Conclusions

The N-body toy model, despite having a simple statement, is a challenging problem from the computational point of view, since the computation cost increases polynomially with the increase in the number of bodies. Even so, simulating it makes it possible to obtain information about the universe on large scales, such as the one studied about arrows of time.

Finally, the model proposed by Shape Dynamics is consistent even with the addition of collisions, which indicates its extensibility for more complex toy models, such as the addiction of inelastic collisions, individual rotation, among others, which we intend to investigate.

References

[1] VOLCHAN, S. Uma Introdução à Mecânica Celeste. Rio de Janeiro: IMPA, 2007.

[2] BARBOUR, J.; KOSLOWSKI, T.; MERCATI, F. Phys. Ver. Lett. **113**, 181101 (2014).

[3] BARBOUR, J.; KOSLOWSKI, T.; MERCATI, F. The solution to the problem of time in shape dynamics. Classical and Quantum Gravity, v. 31, n. 15, p. 155001, 14 jul. 2014.

[4] POTALEJ, O. A. Gravidade-Python. GitHub, 2023: https://github.com/Potalej/gravidade-python.

[5] POTALEJ, O. A. Gravidade-Fortran. GitHub, 2023: https://github.com/Potalej/gravidade-fortran.

[6] ROMA, A. Métodos para a solução numérica de equações diferenciais ordinárias a valores iniciais, 2014. Notas de aula.

[7] HAIRER, E.; WANNER, G.; LUBICH, C. Geometric Numerical Integration: Structure-Preserving Algorithms for Ordinary Differential Equations. Nova Iorque: Springer, 2006.

[8] BOWMAN, J. C; KOTOVYCH, O. An exactly conservative integrator for the N-body problem. Journal of Physics A: Mathematical and General, v. 35, n. 37, p.7849-7863, 5 set. 2002.

[9] BERTSEKAS, D. P. Nonlinear Programming. 3.ed. Nashua: Athena Scientific, 2016.